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EFFECTS OF VARIABLE PHYSICAL PROPERTIES ON HEAT TRANSFER IN FREE CONVECTION AROUND A HORIZONTAL CYLINDER

G. G. Shklover and S. E. Gusev UDC 536.25

Theoretical and experimental studies have been made on the effects of variable viscosity and compressibility on the average heat-transfer coefficient.

General formulas have been recommended [2, 3] in [1] for calculating heat transfer by free convection from a horizontal cylinder; the purpose was to obtain a single formula applicable over wide ranges in the Rayleigh and Prandtl numbers. The variability in the physical properties was incorporated by taking the mean boundary-layer temperature $T_V = 0.5(T_0 + T_C)$ as the definitive quantity. Here we determine the ranges in the physical parameters in which these formulas apply. An approximate method is proposed for incorporating the property variation more precisely.

There are comparatively few papers dealing with the effects of variability in physical properties on heat transfer from horizontal cyliners.

Akagi [4] derived an approximate self-modeling solution and showed that the variability can be incorporated by solving a self-modeling equation system analogous to that for a vertical plate. The solution was derived numerically for an exponential temperature dependence of the dynamic viscosity. The average Nusselt number was derived [4] as

$$\mathrm{Nu} = 0.515\,(\mathrm{Gr}_0\mathrm{Pr}_0)^{0.25}\,(\mu_0/\mu_c)^{0.21}, \tag{1}$$

where $\mathrm{Nu} = \alpha D/\lambda$; $\mathrm{Gr}_0 = g\beta_0 D^3(T_C - T_0)/\nu_0^2$, which applies for $\mathrm{Pr}_0 = 100$ to 10000; $\mu_0/\mu_C = 1$ to 140. The correction factor $(\mu_0/\mu_C)^{0.21}$ can be discarded if one calculates the physical properties from the defining temperature $T_e = T_0 + 0.75(T_C - T_0)$. Formula (1) is close to the experimental one derived in [5].

In [6], measurements for the range $10^2 < \nu_0/\nu_C < 10^4$ gave

$$\mathrm{Nu} = 0.70\,(\mathrm{Gr}_0\mathrm{Pr}_0)^{0.25}\,(\nu_0/\nu_c)^{0.14}. \tag{2}$$

Studies have been made [7, 8] on the effects of simultaneous temperature dependence for the bulk expansion coefficient and kinematic viscosity.

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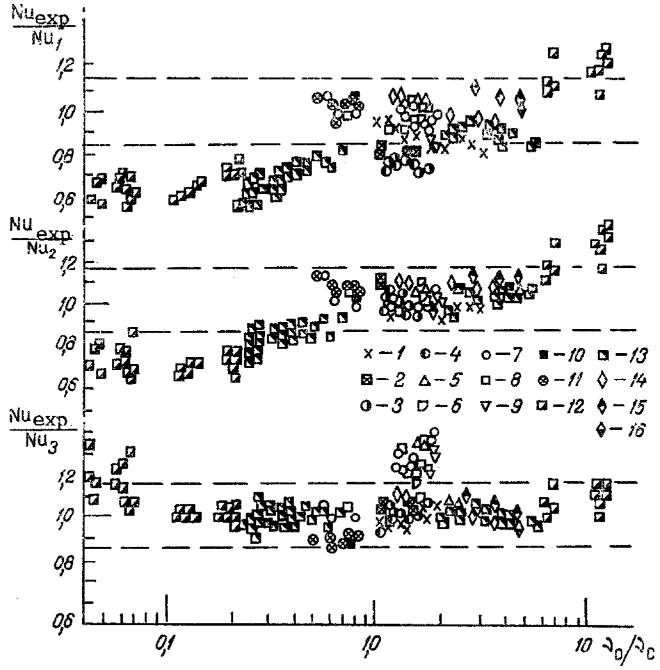

Fig. 1. Comparisons of measured Nu_{exp} with calculated values Nu_i, $i = 1, 2, 3$; Nu_1 – [2], Nu_2 – [3], Nu_3 – [5] as functions of ν_0/ν_c; experimental values, water: 1) [11]; 2) [15]; 3) $T_0 = 30°C$; 4) $T_0 = 40°C$ [10]; 5) D = 6 mm; 6) 8; 7) 10, 8) 12, 9) 16 [9]; air: 10) [5], 11) [12]; 12) MS-20 oil [5]; 13) No. 2 transformer oil [5]; 14) Cs-20 oil; 15) Cs-100; 16) Cs-350 [11].

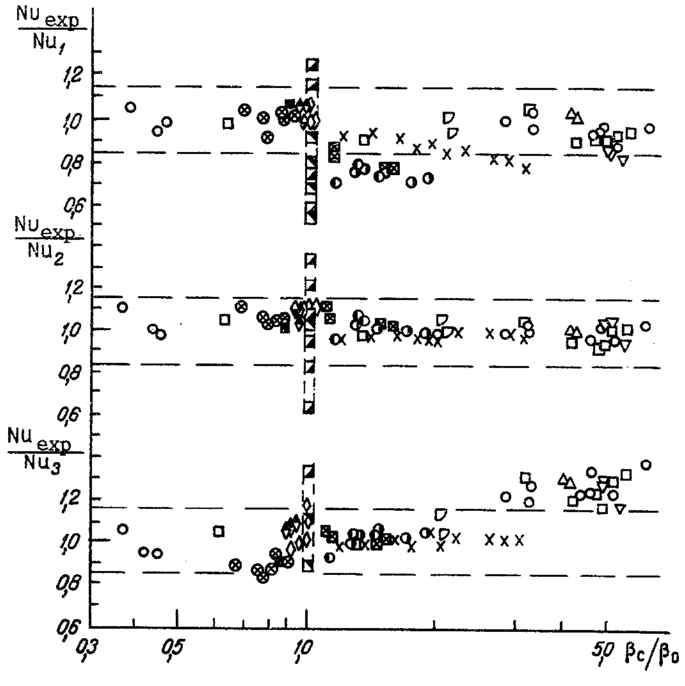

Fig. 2. Dependence on β_c/β_0, symbols as in Fig. 1.

Here a comparison with experiment is used to estimate the errors of certain formulas associated with inadequate incorporation of the property variation. An approximate method is used to derive the analytic form for the correction for simultaneous variation in viscosity and compressibility. The method has been checked by comparison with the exact solution and with published data.

We estimated the accuracy of certain general formulas from papers containing primary experimental data: for water [5, 9, 10, 11], air [12], and oils [5, 11]. Measurements in the laminar range were used: $7.1 \cdot 10^4 \le Ra_V \le 2 \cdot 10^8$. Figures 1 and 2 show the results. The formulas of [2, 3] are based on the defining temperature T_V and describe the measurements satisfactorily if ν is slightly variable if the simplex $N = \nu_0/\nu_c$ is close to one. The error increases to 25% for $N \geq 10$ and to 40% for $N \leq 0.1$. The formula of [5] is based on the defining temperature T_0 and incorporates the variable viscosity via the simplex $(Pr_0/Pr_c)^{0.25}$; it gives a good description of the data for oils, but it diverges from the measurements for water at $B = \beta_c/\beta_0 \geq 4$ by up to 30%. None of these three relationships describes the entire data set as a whole.

We consider a laminar free-convective boundary layer around a horizontal cylinder. The wall temperature T_c and the liquid temperature T_0 far from the wall are constant. The viscosity and compressibility are dependent on temperature:

$$\nu_0/\nu = 1 + (N-1)\Theta^{n_1}, \tag{3}$$

$$\beta/\beta_0 = 1 + (B-1)\Theta^{n_2}. \tag{4}$$

We take D and ν_0/D as the scales for length and velocity and write the equation of motion and the energy equation as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \mathrm{Gr}_0\left[\Theta + \frac{B-1}{n_2+1}\Theta^{n_2+1}\right]\sin 2x + \frac{\partial}{\partial y}\left\langle\frac{1}{[1+(N-1)\Theta^{n_1}]}\frac{\partial u}{\partial y}\right\rangle,$$

$$u\frac{\partial\Theta}{\partial x} + v\frac{\partial\Theta}{\partial y} = \frac{1}{\mathrm{Pr}_0}\frac{\partial^2\Theta}{\partial y^2} \tag{5}$$

with the boundary conditions $u = 0$ at $y = 0$, and δ and $\theta = 1$ at $y = 0$, while $\theta = 0$ at $y = \delta_T$.

The problem is solved via the approximate method of [7]. We take the Prandtl number as fairly large and neglect the inertial terms in the equation of motion. The temperature distribution in the thermal boundary layer δ_T is

$$\Theta = (1 - y/\delta_T)^m. \tag{6}$$

Then we can determine the velocity and the mean heat transfer:

$$\mathrm{Nu} = C(\mathrm{Gr}_0\mathrm{Pr}_0)^{0.25}\,\varepsilon. \tag{7}$$

In the terminology of [13], the method may be called semiintegral, since it uses a preset profile not for the two variables but merely for the temperature. A difference from [13] is that it incorporates the thicknesses of the thermal and dynamic boundary layers δ_T and δ as well as the boundary condition at $y = \delta_T$: in [13], $\partial u/\partial y = 0$, while in our case the condition is derived from that of link-up with the solution in the part $\delta_T \leq y \leq \delta$.

Values of C have been given in [7], with $C = 0.5679$ for $Pr = \infty$, which agrees satisfactorily with experiment for highly viscous liquids. Strictly speaking, $Nu/Ra^{0.25}$ for a horizontal cylinder decreases as Ra increases in the laminar range, and the more so the smaller the Prandtl number, so constant C can give a good description of the heat transfer for finite Pr only for a certain range in the Rayleigh number. Nevertheless, the correction ε arising from the property variability given by the approximate method is shown below to agree well with the exact boundary-layer solution.

Calculations give ε for $m = 2$:

$$\varepsilon = \left\langle \frac{21\sqrt{\mathrm{Pr}_0}}{\left(\sqrt{\mathrm{Pr}_0} + \frac{C_1}{k-3}\right)\left(\sqrt{\mathrm{Pr}_0} - \frac{7}{16}\right)} \left\{\sqrt{\mathrm{Pr}_0}\left[\frac{1}{21} + \frac{C_1}{3(k+3)} + \frac{4C_2}{(l-1)l(l+4)} + \frac{4C_1C_2}{(l-1)l(l+k)}\right] - \frac{1}{48} + \frac{21-5k}{42k(k-3)}C_1 - \frac{C_2}{(l-1)l(l+1)} + \frac{2C_1C_2}{l-1}\left[\frac{7-k}{2(k-3)(l+4)(k+l-3)} - \frac{2}{kl(l+1)}\right] + \frac{3C_1^2}{k^2(k^2-9)} + \frac{12C_1^2C_2}{k(k-3)(l-1)(k+l-3)(k+l)}\right\}\right\rangle^{0.25}, \tag{8}$$

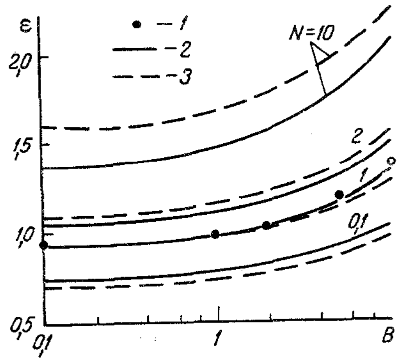

Fig. 3. Correction coefficient for property variability: 1) exact solution, $Pr_0 = 1$; 2 and 3) approximate solutions for $Pr_0 = 1$ and 10^4.

where $C_1 = N - 1$; $C_2 = (B - 1)/2$; $k = 4 + 2n_1$; $\ell = 3 + 2n_2$; ε is dependent on the nature of the liquid (Pr_0, n_1, n_2) and the extent of property variation (N and B). In general, n_1 and n_2 may be dependent on the temperature range. See [9] for a method of determining them, as well as detailed values for certain liquids.

Figure 3 shows ε calculated from (8) for $n_1 = n_2 = 1$. For $B = 4$, $N = 2$, $\varepsilon = 1.29$, which corresponds approximately to Nu_{exp}/Nu_3 (Fig. 2) for $B \geq 4$. Then the discrepancy between measurements and the formula of [5] is due to the latter not fully incorporating the variable β. The formula of [5] fits the data well for $B < 4$.

This method can be applied to transfer at a vertical plate; for the average transfer, one gets an equation analogous to (7) but with a different value of C, while ε takes the form of (8) as for a horizontal cylinder. As a self-modeling solution exists for a vertical plate, one can compare the approximate ε of (8) with the exact value found from the numerical solution of the self-modeling equations:

$$f''' + \frac{3}{4} ff'' - \frac{1}{2}(f')^2 + \Theta + \frac{B-1}{n_2+1}\Theta^{n_2+1} = 0,$$
$$\Theta'' + \frac{3}{4}\mathrm{Pr}_0 f\Theta' = 0. \qquad (9)$$

Equations (9) have been derived by analogy with [14] but on the basis of (4); (9) applies for the following ranges: B from 0.1 to 10, Pr_0 from 1 to 100, and a linear temperature dependence for β ($n_2 = 1$). Figure 3 shows the results. The maximum discrepancy is 0.7% in heat transfer for $Pr_0 = 1$ or 3.6% in the frictional stress. The corresponding figures for $Pr_0 = 100$ are 1.0 and 1.4%.

We also made a comparison with the calculations of [14]; (8) enables one to estimate the contribution from the variability with a maximal error of 6% relative to the numerical calculation of [14] for water with $T_0 = 10\text{-}100°C$, $T_c = 100\text{-}10°C$, so the variability is incorporated quite accurately by this approximate method, which has the advantage that the correction is analytic, so it is not necessary to use laborious numerical calculations on equations such as (9).

Factor ε incorporates the variability in both physical parameters together. It is sometimes useful to estimate the contribution from each separately, so we put ε in the form

$$\varepsilon(N, B) = \varepsilon_1(N)\,\varepsilon_2(B). \qquad (10)$$

The error of (10) is not more than 1.7% for $0.1 \leq N \leq 10$, $0.2 \leq B \leq 5$; from (3) and (4) we have

$$\varepsilon_1 = [\nu_0/\nu(T_1)]^{0.25}, \quad \varepsilon_2 = [\beta(T_2)/\beta_0]^{0.25},$$

where $T_1 = T_0 + \Theta_1(T_c - T_0)$ is the defining temperature for ν, while $T_2 = T_0 + \Theta_2(T_c - T_0)$ is that for β. The dimensionless defining temperatures are

$$\Theta_1=[(\varepsilon_1^4-1)/(N-1)]^{1/n_1},\quad \Theta_2=[(\varepsilon_2^4-1)/(B-1)]^{1/n_2}. \tag{11}$$

Then all the complexity in determining the corrections is transferred to calculating Θ_1 and Θ_2, and if these are known, (7) can be put as

$$\mathrm{Nu}=C(\mathrm{Gr}_*\mathrm{Pr}_*)^{0,25}, \tag{12}$$

where $\mathrm{Gr}_* = g\beta(T_2)(T_c - T_0)D^3/\nu^2(T_1)$, $\mathrm{Pr}_* = \nu(T_1)/\underline{a}$.

Formula (12) is convenient for comparing measurements with calculations for constant properties, since the variability is incorporated by means of defining temperatures, not a correction factor.

When measurements are processed, any change in the temperature used in calculating the thermophysical parameters in the range T_0–T_c can alter the Grashof number considerably. One thus has to choose a defining temperature, i.e., one corresponding to the condition that the equations retain the form applicable to constant physical properties when the properties are variable [15].

Here (11) enables one to calculate these temperatues exactly, which differ for the various parameters. We have $\Theta_2 = 7/27 \approx 0.259$ for $n_2 = 1$ for the bulk expansion coefficient, which corresponds approximately to the mean integral β in the range (T_0, T_v). The Θ_1 for the kinematic viscosity is dependent on Pr_0 and N, and Θ_1 decreases as N increases, while it increases with Pr_0. Values have been given in [7] for m = 2 and $n_1 = 1$. For $\mathrm{Pr}_0 > 10^2$, Θ_1 is close to 0.75. The calculated Θ_1 and Θ_2 correspond well with the empirical values derived from measurements in [16].

A distinct defining temperature is required for each variable parameter; however, it is more convenient to calculate all the parameters for some one temperature, and if this is T_0, one calculates $\mathrm{Nu} = C(\mathrm{Gr}_0\mathrm{Pr}_0)^{0.25}$ with an error of ε. For another calculated temperature $T_{**} > T_0$, the error will be less. We derive it for the case $T_{**} = T_v$.

We apply the above approximate method with ν_v/D instead of ν_0/D as the velocity scale, which gives (13), which is equivalent to (7):

$$\mathrm{Nu}=C(\mathrm{Gr}_v\mathrm{Pr}_v)^{0,25}\varepsilon_v \tag{13}$$

Here ε_v is the correction for the variability in the physical properties for the calculated temperature T_v:

$$\varepsilon_v=[\nu(T_v)/\nu(T_1)]^{0,25}[\beta(T_2)/\beta(T_v)]^{0,25}.$$

One can show that this ε_v in fact corresponds to $\mathrm{Nu}_{exp}/\mathrm{Nu}_{calc}$, as given in Figs. 1 and 2. For example, $\varepsilon_v \approx 1.2$ for MS-20 oil for $N \gtrsim 10$, while $\mathrm{Nu}_{exp}/\mathrm{Nu}_1$ is in the range 1.1-1.3.

The above expressions have been derived for a parabolic temperature profile (m = 2); measurements [12] show that m is dependent on the temperature range and on the x coordinate. In all cases, m lies in the range $2 \le m \le 4$, and the corresponding ε varies by not more than 3% in the ranges $0.1 \le N \le 10$, $0.1 \le B \le 10$, i.e., ε is only slightly dependent on the type of profile in (6), so (8) can be recommended for practical calculations.

The formulas in [2, 3] and (8) are fairly cumbersome. As there is only a slight dependence on the Rayleigh number for $\mathrm{Nu}/\mathrm{Ra}^{0.25}$ in the boundary-layer state, approximate calculations can be based on a formula derived from measurements by least squares:

$$\mathrm{Nu}_0=0.51\,\mathrm{Ra}_0^{0,25}\bar{\varepsilon}(\nu_0/\nu_c)^{0,10}. \tag{14}$$

Here $\bar{\varepsilon}$ is the correction factor for variability in ν and β calculated from the simplified formula

$$\bar{\varepsilon}=\left\langle\frac{1}{1+\frac{C_1}{3}}\left(1+\frac{76}{81}C_1+\frac{14}{27}C_2+\frac{917}{1782}C_1C_2+\frac{28}{243}C_1^2+\frac{7}{99}C_1^2C_2\right)\right\rangle^{0,25},$$

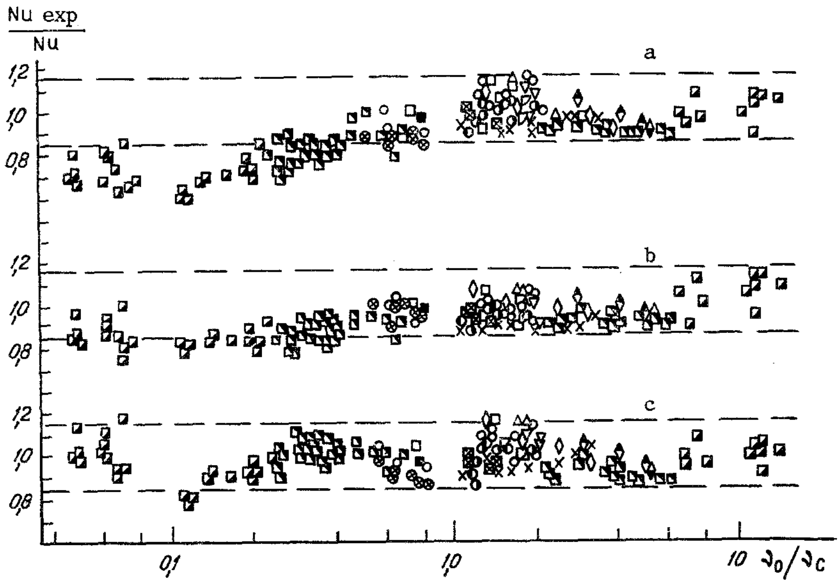

Fig. 4. Comparison of measurements with calculations from (7): a) $n_1 = 1$; b) $n_1 \neq 1$; c) calculation from (14). Symbols as in Fig. 1.

where $C_1 = N - 1$, $C_2 = (B - 1)/2$; $\bar{\varepsilon}$ corresponds to (8) for $Pr_0 = 1$, $n_1 = n_2 = 1$.

Figure 4c shows calculations from (14) together with data processed from (7). The calculated Nu in Fig. 4a have been derived on the assumption of a linear temperature dependence for $1/\nu$ and β ($n_1 = n_2 = 1$). For $N \leq 0.4$, the calculated values are overestimates by 30%, which is because $1/\nu$ for MS-20 oil does not show linear behavior.

The refined description gives $n_1 < 1$ for $N < 1$ (Fig. 4b). For $N < 0.4$, the error is on average 15%, which is comparable on the one hand with the error of experiment and on the other with the error in describing $1/\nu$ as a function of Θ via (3). Formula (14) describes the measurements for the ranges $Ra_0 = 2 \cdot 10^4 - 10^8$, $Pr_0 = 0.7 - 3 \cdot 10^4$, $N = 0.04 - 12$, $B = 0.4 - 6$ with a maximum error of $\pm 15\%$. The standard deviation is 7%. This formula can be recommended for approximate calculations.

NOTATION

α, heat-transfer coefficient, $W/m^2 \cdot {}^\circ C$; T_C, wall temperature, °C; T_0, liquid temperature far from wall, °C; ν, kinematic viscosity, m^2/sec; β, bulk expansion coefficient, 1/°C; D, cylinder diameter, m; x and y, dimensionless coordinates; u and v, dimensionless velocities of liquid along x and y axes corrspondingly; $\Theta = (T - T_0)/(T_C - T_0)$, dimensionless temperature; $Pr = \nu/a$, Prandtl number; $Gr = g\beta(T_C - T_0)D^3/\nu^2$, Grashof number; $Nu = \alpha D/\lambda$, Nusselt number; $Ra = GrPr$, Rayleigh number; f, dimensionless current function [14]; ε_1, correction factor for variability of ν; ε_2, correction factor for variability of β; T_1 and T_2, defining temperatures for ν and β, °C; Θ_1 and Θ_2, dimensionless defining temperature for ν and β. Subscripts: 0, v, and c, relate to the temperatures T_0, T_V, and T_C correspondingly.

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HEAT TRANSFER WITH CONDENSATION ON MESH CAPILLARY STRUCTURES OF HEAT PIPES

Z. Sh. Semerkhanov, G. F. Smirnov, and
B. A. Afanas'ev

UDC 536.421.4

The authors present results of an experimental investigation of heat transfer with condensation of water vapor on the capillary structures of heat pipes. They compare the test data with theory.

The thermal resistance of low-temperature heat pipes depends on the intensity of heat transfer in the condensation section more strongly, the less is the heat removal area compared with the surface area of the heat supply zone. The need to reduce the mass and size of heat exchange equipment has led to the situation that in heat pipes with minimum possible length of the condensation section the total thermal resistance of the pipe is roughly the same as the thermal resistance of the heat removal section. Therefore for this type of heat pipe it is especially important to have: 1) well-founded physical ideas as to the heat removal mechanism for different conditions of condensation on the capillary surfaces of the heat pipe; 2) reliable knowledge of the dependence of the heat removal intensity on the basic independent factors.

The presently available experimental data on heat removal with condensation in capillary structures has been obtained from tests on heat pipes (e.g., [1-5]). In most cases the test results suggest a thermal resistance with condensation in the form

$$R_c = \delta_\phi/\lambda_e. \tag{1}$$

It is noted in [1] that on the surface of a metal fiber capillary structure with condensation there is a liquid film of thickness on the order of 10 μm. It is suggested that the thickness of this film is constant and does not depend on the main regime and geometrical parameters.

Reference [2] suggested a method of calculating R_c, using the heat transfer coefficient with condensation α_c, determined from the correlation equations. But here no basis was given for choice of the correlation or the structure of the correlation equations.

It should be noted that in tests on heat pipes one can have too little and too much heat transfer agent, leading to a deviation of the heat transfer coefficients with condensation from the nominal value.

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